Int. J. Heat Mass Transfer. Vol. 5, pp. 639–660. Pergamon Press 1962. Printed in Great Britain.

CONVECTIVE AND RADIANT HEAT TRANSFER FOR FLOW OF A TRANSPARENT GAS IN A TUBE WITH A GRAY WALL

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(*Received* 12 *June* 1961 *and in revised form* 26 *December* 1961)

Abstract—The influence of thermal radiation exchanges inside of a circular tube combined with convective heat transfer is studied analytically. The tube has a uniform heat flux imposed at the wall by external means such as uniform electrical heating, and the wall temperature distribution is found from the analysis. The gas flowing within the tube is transparent to radiation and hence does not influence the radiation exchange between elements of the internal tube surface. The inside of the tube wall is assumed to be a diffuse gray surface, and the outside is perfectly insulated. The solutions are governed by seven independent parameters such as the wall emissivity, inlet gas temperature, and length–diameter ratio of the tube. Numerical examples are given to show the influence of these parameters and to demonstrate how radiation alters the wall temperature distribution that would exist for convection alone.

NOMENCLATURE

c_p, specific heat of fluid;
D, tube diameter;
F, geometric configuration factor for radiation from an element on the tube wall to the circular opening at the end of the tube;
H, dimensionless heat-transfer coefficient, $(h/q)(q/\sigma)^{1/4}$;
$H_{\exp}$, heat-transfer coefficient which includes both convection and radiation;
h, convective heat-transfer coefficient;
K, geometric configuration factor between elements on inside of tube wall;
k, thermal conductivity of gas;
L, length of tube;
l, dimensionless length, L/D;
Nu, Nusselt number, hD/k;
Pr, Prandtl number, $c_p\mu/k$;
q, heat added per unit area at tube wall;
q_i, total incoming radiation per unit area to a surface element;
q_0, total outgoing radiation per unit area from a surface element;
Re, Reynolds number, $u_m D\rho/\mu$;
S, Stanton number, $4\,Nu/RePr = 4h/u_m\rho c_p$;
T, temperature;
t, dimensionless temperature, $(\sigma/q)^{1/4}T$;
u_m, mean gas velocity;
X, axial length co-ordinate measured from tube entrance;
x, dimensionless co-ordinate, X/D;
ϵ, emissivity of surface;
μ, viscosity of gas;
Ξ, dummy integration variable;
ξ, dimensionless variable, Ξ/D;
ρ, density of gas;
σ, Stefan–Boltzmann constant.

Subscripts

e, exit end of tube;
g, gas;
i, inlet end of tube (except in symbol q_i);
r, reservoir;
w, inside surface of tube wall.

INTRODUCTION

HEAT transfer by forced convection to a gas flowing in a tube has received detailed study in the literature, but little consideration has been given to the added effects caused when thermal radiation is also present. The radiation exchanges become especially important at the high temperature levels encountered in advanced types of power-plants such as the nuclear rocket. In some instances, the radiation will impose an additional heat load on a part which is to be kept cool, and hence this exchange must be estimated when the cooling requirements are computed. In other cases, the radiation will help reduce the temperature of a region operating at a high temperature.

This paper is concerned with the energy exchanges that occur inside a tube when heat is being transferred by radiant interchange between elements on the tube wall and by forced convection to a gas which does not absorb or emit radiation. The tube is heated by an externally applied uniform heat flux imposed at the wall. This heat flux would usually be caused by a uniform heat generation in the wall, such as in the channel of a nuclear reactor or in a forced-convection experiment with an electrically heated tube. The gas is heated only by convection from the wall, and the amount of this heating depends on the difference between the local wall and bulk gas temperatures and on the convective heat-transfer coefficient. The thermal radiation between elements of the wall inside the tube alters the wall temperature distribution from the pure convection case and this in turn influences the gas temperature variation along the tube length.

Some previous work on combined convection and radiation has been given [1–3]. The latter two references deal with flow in a tube such as treated in the present paper, but are limited to the condition that the radiating surface is black ($\epsilon = 1$). In [2], one numerical case was obtained for a short tube by division of the tube length into several isothermal sections, after which a heat balance on each of these regions was taken. This resulted in a set of nonlinear algebraic equations which were solved for the wall temperature in each isothermal zone. In [3] the problem was examined in greater detail and numerical solutions were carried out to show the effect of each of the independent parameters. Two methods of solution were employed. In one, a separable kernel was used to approximate the geometrical configuration factor for radiation between elements of the internal tube surface. With this approximation, the integral equation could be transformed into a second-order ordinary differential equation which was integrated numerically. In the second method of solution, the integral equation for the temperature distribution was placed in finite-difference form. This gave a set of nonlinear algebraic equations which were solved by using the Newton–Raphson method. The second method was used principally as a check on the first, and good agreement was obtained. In [4] the special case was considered where the convection is negligible compared with the radiation, and wall temperature distributions were found for both black and diffuse gray radiation.

The present paper provides the additional analysis necessary to extend [3] to include diffuse gray radiation. The resulting equations reduce to those in [3] as a special case when $\epsilon = 1$. The gray-wall assumption is made, which states that the emissivity and absorptivity are independent of wavelength and are equal. They still could be a function of wall temperature, but it is assumed that they are constant over the wall temperature range for any numerical case (in some references the independence with temperature is also included in the gray assumption). Both of the methods given in [3] can be used to solve the gray-wall equations. However, the direct numerical solution of the integral equation became quite involved and hence using a separable kernel was the most convenient of the two methods. The approximations made in the separable-kernel method are confined to the radiation terms. For gray walls, the radiation exchanges would be expected to contribute a smaller fraction of the total heat transfer than for black walls, so the error in the approximations is decreased when the emissivity is less than 1. Hence the separable-kernel method should be satisfactory for the gray-wall problem, since in [3] it was shown to give reasonable results for the black-wall case.

ANALYSIS

The system to be analyzed is shown schematically in Fig. 1. A transparent gas at a specified inlet temperature $T_{g,i}$ flows into the tube and is heated to an average exit temperature $T_{g,e}$. A uniform heat flux q is supplied to the tube wall by external means (e.g. electrical heating, nuclear fission), and the outside surface of the tube is assumed insulated. Each end of the tube is exposed to an outside environment or reservoir which is at a specified temperature $T_{r,i}$ at the inlet end of the tube and at $T_{r,e}$ at the exit end. The inside of the tube wall is a diffuse gray surface with an emissivity ϵ. It is assumed that there is no axial conduction in the tube wall or in the gas and that the convective heat-transfer coefficient h is a constant throughout the tube.

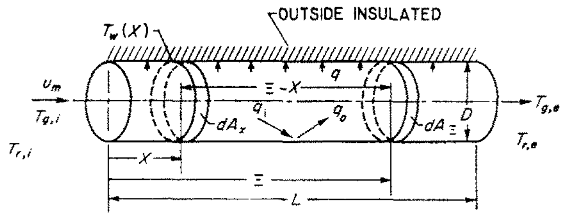

FIG. 1. Circular tube geometry.

Energy balance

The energy balance including radiation exchanges will be derived by use of Poljak's net-radiation method described in [5]. According to this method, we let q_0 be the rate of outgoing radiation from the surface, which is composed of the direct emission plus the reflection of incoming radiation. The rate of incoming radiation to an element on the surface is designated as q_i. Then for an element on the tube surface the energy balance can be formed as

$$q + q_i = q_0 + h[T_w(X) - T_g(X)]. \quad (1)$$

The terms on the left are the externally imposed heat input q and the incoming radiation. On the right there is the outgoing radiation and the heat leaving by convection from the wall to the gas. $T_w - T_g$ is the local difference between the wall temperature and bulk gas temperature. The imposed heat flux q and the convective heat-transfer coefficient h are assumed independent of axial position, although variations along the tube could be accounted for in the analysis. The radiation terms are now considered in detail.

The radiative heat flux q_0 leaving a surface element is composed of direct emission, $\epsilon\sigma T_w^4$, and of reflected radiation which is $(1 - \epsilon)$ times the incoming radiation:

$$q_0 = \epsilon\sigma T_w^4 + (1 - \epsilon)q_i. \quad (2)$$

The incoming radiative heat flux q_i is composed of two types of terms, the radiation coming from the reservoirs at the ends of the tube and the radiation arriving as a result of the outgoing radiation from the other elements on the internal tube surface. These quantities can be written as

$$q_i = \sigma T_{r,i}^4 F(x) + \sigma T_{r,e}^4 F(l - x) + \int_0^x q_0(\xi)K(x - \xi)\,d\xi + \int_x^l q_0(\xi)K(\xi - x)\,d\xi. \quad (3)$$

The function $F(x)$ is the geometric configuration factor for radiation from an element on the tube wall at location x to the circular opening at the inlet end of the tube. This factor is given by [2]

$$F(x) = \frac{x^2 + 1/2}{(x^2 + 1)^{1/2}} - x \qquad x \geq 0. \quad (4)$$

It has been assumed that the reservoirs can be represented by black circular disks at the end openings of the tube. This would be a good approximation in many instances when the reservoirs are chambers with dimensions larger than the tube diameter, which causes the reservoirs to behave like black-body cavities when viewed through the end openings. The function $K(z)$ is the configuration factor between two rings on the inside surface of the tube a distance z apart and is given by [2]

$$K(z) = 1 - \frac{z^3 + 3z/2}{(z^2 + 1)^{3/2}} \qquad z \geq 0. \quad (5)$$

Equation (3) is substituted into equation (1) to eliminate q_i with the result

$$h[T_w(x) - T_g(x)] + q_0 = q + \sigma T_{r,i}^4 F(x) + \sigma T_{r,e}^4 F(l - x) + \int_0^x q_0(\xi)K(x - \xi)\,d\xi + \int_x^l q_0(\xi)K(\xi - x)\,d\xi. \quad (6)$$

This equation has three unknowns q_0, T_w and T_g, so two additional relations are needed. To relate q_0 and T_w, equations (1) and (2) are combined to eliminate q_i with the result

$$q_0 = \frac{(1 - \epsilon)}{\epsilon}[h(T_w - T_g) - q] + \sigma T_w^4. \quad (7)$$

To relate T_g and T_w, an additional heat balance is written for the flowing gas. Since the gas is transparent to radiation, the only heat transferred to it is by convection from the wall. For a cylindrical volume element of length $\mathrm{d}X$ and diameter D, the heat transferred is

$$h\pi D\,\mathrm{d}X\,[T_w(X) - T_g(X)].$$

This is equal to the net heat carried out of the volume element by the flowing gas, which is

$$u_m \frac{\pi D^2}{4} \rho c_p \frac{\mathrm{d}T_g}{\mathrm{d}X}\,\mathrm{d}X.$$

The mean fluid velocity u_m is assumed constant so that kinetic energy changes of the gas are neglected. These two quantities are equated and the result is rearranged into the form

$$\frac{\mathrm{d}T_g}{\mathrm{d}x} = S\,[T_w(x) - T_g(x)] \tag{8}$$

where S is the Stanton number, $4h/u_m \rho c_p$, and $x = X/D$.

Equations (6–8) form a set of three simultaneous equations for the three unknowns T_w, T_g and q_0. Only the first two of these are of physical interest, so q_0 will be eliminated as the analysis proceeds.

Transformation to a differential equation

The integral equation (6) can be reduced to a differential equation by use of an approximate separable function for the geometric kernel K. The approximation used in [3] and [4] was the exponential function

$$K(z) \simeq \mathrm{e}^{-2z} \qquad z \geq 0. \tag{9}$$

This gave very good results for short tubes with length–diameter ratios less than about 10, as shown in [3]. For longer tubes, it was pointed out in [6] that this approximation becomes less accurate and $K(z)$ is better approximated by a sum of exponential terms. However, in the present problem, for long tubes convection generally becomes more dominating, as shown in Fig. 12 (see p. 658), and hence the approximation for the radiation exchange does not have to be as precise. For this reason the approximation in equation (9) appears adequate for the combined radiation and convection cases treated here.

Equation (9) is substituted into equation (6) and the resulting equation is differentiated twice to give

$$\left.\begin{aligned} h\left(\frac{\mathrm{d}^2T_w}{\mathrm{d}x^2} - \frac{\mathrm{d}^2T_g}{\mathrm{d}x^2}\right) + \frac{\mathrm{d}^2q_0}{\mathrm{d}x^2} &= \sigma T^4_{r,i}\frac{\mathrm{d}^2F(x)}{\mathrm{d}x^2} + \sigma T^4_{r,e}\frac{\mathrm{d}^2F(l-x)}{\mathrm{d}x^2} \\ &\quad + 4\left[\frac{1}{\mathrm{e}^{2x}}\int_0^x q_0(\xi)\,\mathrm{e}^{2\xi}\,\mathrm{d}\xi \right. \\ &\quad \left. + \mathrm{e}^{2x}\int_x^l q_0(\xi)\,\mathrm{e}^{-2\xi}\,\mathrm{d}\xi\right] - 4q_0. \end{aligned}\right\} \tag{10}$$

The integrals in equation (10) are eliminated by subtracting four times equation (6), which gives

$$\begin{aligned} h\left(\frac{\mathrm{d}^2T_w}{\mathrm{d}x^2} - \frac{\mathrm{d}^2T_g}{\mathrm{d}x^2} - 4T_w + 4T_g\right) &+ \frac{\mathrm{d}^2q_0}{\mathrm{d}x^2} = -4q + \sigma T^4_{r,i}\left[\frac{\mathrm{d}^2F(x)}{\mathrm{d}x^2} - 4F(x)\right] \\ &+ \sigma T^4_{r,e}\left[\frac{\mathrm{d}^2F(l-x)}{\mathrm{d}x^2} - 4F(l-x)\right]. \end{aligned} \tag{11}$$

The quantity $\mathrm{d}^2q_0/\mathrm{d}x^2$ can be eliminated by means of the expression obtained by differentiating equation (7) twice:

$$\begin{aligned} \frac{\mathrm{d}^2q_0}{\mathrm{d}x^2} = \frac{h(1-\epsilon)}{\epsilon}\left(\frac{\mathrm{d}^2T_w}{\mathrm{d}x^2} - \frac{\mathrm{d}^2T_g}{\mathrm{d}x^2}\right) \\ + 4\sigma T_w^3\frac{\mathrm{d}^2T_w}{\mathrm{d}x^2} + 12\sigma T_w^2\left(\frac{\mathrm{d}T_w}{\mathrm{d}x}\right)^2. \end{aligned} \tag{12}$$

To eliminate the second derivative of the gas temperature, equation (8) is differentiated and then equation (8) is substituted in the result to remove $\mathrm{d}T_g/\mathrm{d}x$. This gives

$$\frac{\mathrm{d}^2T_g}{\mathrm{d}x^2} = S\left[\frac{\mathrm{d}T_w}{\mathrm{d}x} - S(T_w - T_g)\right]. \tag{13}$$

The result of substituting equations (12) and (13) into equation (11) is the following differential equation which has been placed in dimensionless form:

$$\begin{aligned} \frac{\mathrm{d}^2t_w}{\mathrm{d}x^2}\left(4t_w^3 + \frac{H}{\epsilon}\right) + 12t_w^2\left(\frac{\mathrm{d}t_w}{\mathrm{d}x}\right)^2 - \frac{SH}{\epsilon}\frac{\mathrm{d}t_w}{\mathrm{d}x} \\ + t_w\left(\frac{S^2H}{\epsilon} - 4H\right) = t_g\left(\frac{S^2H}{\epsilon} - 4H\right) - 4 \\ + t^4_{r,i}\left[\frac{\mathrm{d}^2F(x)}{\mathrm{d}x^2} - 4F(x)\right] \\ + t^4_{r,e}\left[\frac{\mathrm{d}^2F(l-x)}{\mathrm{d}x^2} - 4F(l-x)\right]. \end{aligned} \tag{14}$$

Equations (14) and (8) are two simultaneous equations for the wall and gas temperatures along the tube. They could be solved by numerical integration in their present form, but before this is done an additional simplification is made. It was shown in [3] and [4] that the configuration factor F could be approximated with good accuracy by the exponential function

$$F(x) \simeq \frac{e^{-2x}}{2}. \tag{15}$$

With this function, the quantity $[d^2F/dx^2 - 4F]$ is equal to zero and hence the reservoir temperatures $t_{r,i}$ and $t_{r,e}$ do not appear in the differential equation (14). The reservoir temperatures will enter the solution through the boundary conditions. To summarize: the final differential equations to be solved simultaneously are

$$\frac{d^2t_w}{dx^2}\left(4t_w^3 + \frac{H}{\epsilon}\right) + 12t_w^2\left(\frac{dt_w}{dx}\right)^2 - \frac{SH}{\epsilon}\frac{dt_w}{dx} + t_w\left(\frac{S^2H}{\epsilon} - 4H\right) = t_g\left(\frac{S^2H}{\epsilon} - 4H\right) - 4 \tag{16a}$$

$$\frac{dt_g}{dx} = S(t_w - t_g). \tag{16b}$$

Boundary conditions

Equation (16b) is a first-order equation and hence requires only one boundary condition. The condition is that at the inlet of the tube the gas temperature has a specified value $t_{g,i}$:

$$t_g = t_{g,i} \quad \text{at} \quad x = 0. \tag{17}$$

Equation (16a) is a second-order equation and requires two boundary conditions. These are found by use of the approximations for K and F in the integral equation (6) and then evaluation of it at $x = 0$ and $x = l$. At $x = 0$ this gives:

$$h[T_w(0) - T_g(0)] + q_0(0) = q + \frac{\sigma T_{r,i}^4}{2} + \frac{\sigma T_{r,e}^4 e^{-2l}}{2} + \int_0^l q_0(\xi)\, e^{-2\xi}\, d\xi \tag{18}$$

while at $x = l$:

$$h[T_w(l) - T_g(l)] + q_0(l) = q + \frac{\sigma T_{r,i}^4 e^{-2l}}{2} + \frac{\sigma T_{r,e}^4}{2} + \frac{1}{e^{2l}}\int_0^l q_0(\xi)\, e^{2\xi}\, d\xi. \tag{19}$$

The outgoing radiation q_0 is eliminated by means of equation (7), and part of the integration can be carried out analytically. This substitution was carried out only for the boundary condition at $x = l$, because the condition at $x = 0$ is used in a different form as described in the next paragraph. The final form of equation (19) with q_0 eliminated and the result rearranged into dimensionless form is

$$\frac{(1-\epsilon)}{2\epsilon}(1 + e^{-2l}) + 1 + t_{r,i}^4\frac{e^{-2l}}{2} = \frac{H}{\epsilon}\left[t_w(l) - t_g(l)\right] + t_w^4(l) - \frac{t_{r,e}^4}{2} - \int_0^l \left[\frac{(1-\epsilon)}{\epsilon}H(t_w - t_g) + t_w^4\right] e^{-2(l-x)}\, dx. \tag{19a}$$

In order to begin a numerical forward integration of equations (16a) and (16b) it is necessary to know at $x = 0$ the values of the wall and gas temperatures and the first derivative of the wall temperature. The inlet gas temperature is specified for any particular problem, but the wall temperature at $x = 0$ is unknown and will have to be found by trial and error, as discussed later. For determination of the initial wall temperature derivative, equation (6) with the approximate K and F from (9) and (15) is differentiated once and evaluated at $x = 0$.

$$h\left(\left.\frac{dT_w}{dx}\right|_0 - \left.\frac{dT_g}{dx}\right|_0\right) + \left.\frac{dq_0}{dx}\right|_0 = -\sigma T_{r,i}^4 + \sigma T_{r,e}^4 e^{-2l} + 2\int_0^l q_0(\xi)\, e^{-2\xi}\, d\xi.$$

The integral on the right is eliminated by means of the boundary condition at $x = 0$, equation (18); then $q_0(0)$ is eliminated by means of equation (7), dq_0/dx at $x = 0$ by the first derivative of equation (7), and dT_g/dx by equation (8).

The resulting equation is solved for the initial wall temperature derivative which has the final dimensionless form

$$\left.\frac{dt_w}{dx}\right|_{x=0} = \frac{1}{H + 4\epsilon t_w^3(0)} \left\{ H(S+2)\,[t_w(0) - t_{g,i}] + 2\epsilon t_w^4(0) - 2 - 2\epsilon t_{r,i}^4 \right\}. \quad (20)$$

Since the boundary condition at $x = 0$, equation (18), has been used to obtain this relation, this boundary condition will always be fulfilled at the start of the numerical integration. Hence the only boundary condition which remains to be satisfied is equation (19a), and this will be fulfilled when the proper value for $t_w(0)$ is found.

Numerical solution

To carry out a solution it is first necessary to choose a value for each of the seven independent parameters that are involved. These are H, S, $t_{r,i}$, $t_{r,e}$, $t_{g,i}$, l and ϵ. The simultaneous equations (16) for the wall and gas temperatures were programmed for solution on a digital computer with a forward integration technique. This has been described in [3] where the increment sizes required and the sensitivity of the solution are discussed for a similar type of equation. The same computational procedure was used in the present paper and it is not worthwhile to repeat the details here. After assigning values to the parameters, the next step is to estimate a value for the wall temperature at the beginning of the tube $t_w(0)$. From this value the wall temperature derivative at $x = 0$ can be found from equation (20). With these initial values, the forward integration of the differential equations can be carried out, and the solution is then substituted into the boundary condition equation (19a). If (19a) is not satisfied, then a new trial for $t_w(0)$ is made and this process is continued until the desired solution is found.

Over-all heat balance

An additional check of each numerical solution was made by making sure that it always satisfied an over-all heat balance. The individual terms in the heat balance also provide some useful information that will be tabulated later. The balance was derived from the following terms: The heat imposed at the tube wall is $q\pi DL$. The heat carried away by the gas is $\rho u_m c_p(\pi D^2/4)(T_{g,e} - T_{g,i})$. The heat radiated in from the reservoirs at the ends of the tube is

$$\sigma\pi D \left[T_{r,i}^4 \int_0^L F\left(\frac{X}{D}\right) dX + T_{r,e}^4 \int_0^L F\left(\frac{L}{D} - \frac{X}{D}\right) dX \right]$$

and the heat radiated from the tube surface out through the ends of the tube is

$$\pi D \int_0^L q_0 \left[F\left(\frac{X}{D}\right) + F\left(\frac{L}{D} - \frac{X}{D}\right) \right] dX.$$

The heat balance is formed from these terms and, after substitution of the approximate exponential functions for K and F, the integrals of the F terms are carried out analytically. The heat balance is then placed in the dimensionless form

$$l + \frac{H}{S} t_{g,i} + \frac{(1 - e^{-2l})}{4} t_{r,i}^4 = -\frac{(1 - e^{-2l})}{4} t_{r,e}^4 + \frac{H}{S} t_{g,e} + \frac{1}{2}\int_0^l \frac{q_0}{q} \left[e^{-2x} + e^{-2(l-x)} \right] dx.$$

The outgoing radiation q_0 is eliminated by substitution of equation (7), and two of the terms under the integral can then be integrated analytically. After rearrangement, the heat balance takes the final form

$$l + \frac{H}{S} t_{g,i} + \frac{t_{r,i}^4}{4}(1 - e^{-2l}) + \frac{(1-\epsilon)}{2\epsilon}(1 - e^{-2l}) = -(1 - e^{-2l})\frac{t_{r,e}^4}{4} + \frac{H}{S} t_{g,e} + \frac{1}{2}\int_0^l \left[\frac{(1-\epsilon)}{\epsilon} H(t_w - t_g) + t_w^4 \right] [e^{-2x} + e^{-2(l-x)}]\, dx. \quad (21)$$

INDEPENDENT PARAMETERS

Before the results of the analysis are discussed it is desirable to review briefly the seven independent parameters that must be chosen for each numerical case.

(1) ϵ is the emissivity of the wall which is assumed to be a diffuse gray surface. The variation of emissivity with temperature is assumed negligible over the range of temperatures encountered in each solution. Examples are given for ϵ ranging from 1·0 for a black wall to 0·01 for a highly reflecting wall.

(2) l is the tube length expressed in terms of tube diameters. This geometry factor has an important effect on the wall temperature distributions as it determines how well the internal surface of the tube can exchange radiation directly with the external reservoirs.

(3) H is the parameter $(h/q)(q/\sigma)^{1/4}$. When q has a fixed value, H is directly proportional to the convective heat-transfer coefficient.

(4) The specified temperature of the gas entering the tube is expressed in terms of the parameter $t_{g,i} = T_{g,i}(\sigma/q)^{1/4}$. When $t_{g,i}$ is increased, the temperature level in the system is raised and this increases the radiation exchanges.

(5) The inlet and exit reservoirs are expressed in terms of the dimensionless groups $t_{r,i} = T_{r,i}(\sigma/q)^{1/4}$ and $t_{r,e} = T_{r,e}(\sigma/q)^{1/4}$. When q is a specified constant, the dimensionless groups are directly proportional to the reservoir temperatures.

(6) The parameter S is the Stanton number $4Nu/RePr = 4h/u_m\rho c_p$. For a fixed q and h, an increase in S, caused for example by a decreased flow rate, increases the axial gas temperature gradient along the tube.

RADIATION CORRECTION FACTOR FOR HEAT-TRANSFER COEFFICIENT

In addition to the foregoing parameters there is one more quantity which should be discussed before the specific results of the analysis are presented. This deals with the interpretation of forced-convection experiments in electrically heated tubes to determine convective heat-transfer coefficients for high-temperature conditions. To determine the local heat-transfer coefficient, measurements would be made of local wall and gas temperatures in the tube. Then with q known, the experimental heat-transfer coefficient would be $h_{\exp} = q/(T_w - T_g)$. This is not a convective coefficient because it is the result of both radiation and convection processes within the tube. There are some additional effects such as the axial heat conduction in the tube wall which would contribute to $h_{\exp}$, but since these have not been included in the present analysis they cannot be discussed here. To obtain h for convection alone it is necessary to correct $h_{\exp}$ to account for the radiation exchanges, and a theoretical radiation correction factor can be derived from the present analysis. This is given as the ratio $h/h_{\exp}$, which can be multiplied by the experimental coefficient for combined radiation and convection to yield the coefficient for convection alone. The ratio is equal to $h/h_{\exp} = (h/q)(T_w - T_g)$, or in dimensionless form $H/H_{\exp} = H(t_w - t_g)$. This was evaluated from the wall and gas temperature distributions found in the analysis, and results will be given in the sections that follow.

LIMITING CASES OF PURE CONVECTION AND RADIATION AND THEIR SIGNIFICANCE

There are two limiting cases which are useful for comparison with the present results. One is the result for pure convection which is achieved when the radiation effects are very small. This limit would be expected as the temperature level of the system diminishes, as the emissivity of the surface decreases toward zero, or as the convective heat-transfer coefficient becomes very large. For pure convection with fully developed flow and heat transfer in a tube where there is a uniform heat addition at the wall, both the gas and wall temperatures rise linearly along the tube length. The gas temperature variation can be found from a heat balance on the flowing gas, which states that

$$q\pi D\,\mathrm{d}X = u_m \frac{\pi D^2}{4} \rho c_p \frac{\mathrm{d}T_g}{\mathrm{d}X}\,\mathrm{d}X.$$

This is integrated to determine T_g, and the result can be placed in the dimensionless form

$$t_g = \frac{S}{H} x + t_{g,i}. \tag{22}$$

The local wall temperature differs from the local gas temperature by a constant which is found from the convective heat-transfer coefficient,

$$q = h(T_w - T_g) \text{ or } T_w = \frac{q}{h} + T_g.$$

After substitution of equation (22) for the gas temperature, the wall temperature can be placed in the dimensionless form,

$$t_w = \frac{S}{H} x + \frac{1}{H} + t_{g,\,i}. \qquad (23)$$

The limit of pure radiation is reached when convection becomes small compared with the net radiation exchange. This will occur, for example, when the convective heat-transfer coefficient is small or when the temperature level in the system is high so that the radiation exchanges are large. It was shown in [4] that, by use of the net radiation method, the solution for a diffuse gray wall could be easily found when the result for a black wall was known. This led to the following solution for the wall temperature obtained by using the approximate separable kernel method:

$$t_w = \left[\frac{1}{\epsilon} + t_{r,\,e}^4 + l + 2(xl - x^2) + (t_{r,\,i}^4 - t_{r,\,e}^4)\frac{(\frac{1}{2} + l - x)}{l + 1}\right]^{1/4}. \qquad (24)$$

Because of the approximations used in [4], this relationship becomes less accurate for long tubes ($L > 10$), but still should indicate the general trend of the pure radiation temperature levels.

These limiting cases can be used to obtain some insight into the nature of some of the solutions that follow. This interpretation applies when both reservoir temperatures are lower than the wall temperatures so that there is no net radiation to the wall from the environments. In this instance, all of the heat that leaves the tube by convection or radiation arises from the q imposed at the wall. The wall temperatures required to dissipate the heat by pure convection or by pure radiation are obtained from equations (23) and (24). The limiting case that gives the lowest temperatures corresponds to the mechanism by which the heat can be transferred from the tube most efficiently. Since heat can be transferred by both mechanisms more effectively than by either process alone, the wall-temperature solution will fall below the envelope formed by the pure-convection and pure-radiation curves. If one limiting curve falls considerably below the other, the exchange mechanism for the lower curve is much more efficient than the other process and it will dominate the heat transfer. In this instance the wall temperatures will be slightly below the lower curve. However if the limiting curves fall close together this means that both exchange mechanisms are of comparable efficiency and the wall temperatures for combined radiation and convection will fall considerably below both limiting curves. This discussion will be illustrated in the sections that follow. It can also be used to interpret how the results would change if the convective heat-transfer coefficient were a function of X, such as in a thermal-entrance region. There, the convective-wall-temperature curve would be lower owing to higher h-values. The wall temperatures for combined radiation and convection would be expected to fall below this convection curve.

RESULTS FOR SHORT TUBES ($L/D = 5$)

Solutions for various values of the parameters were obtained for a short tube having a length–diameter ratio of 5. The values of the parameters were chosen to show the behavior of the system over a physically realistic range of variables. For short tubes, the entire inside surface of the tube can readily exchange heat by radiation directly with the external environment, and hence the radiation effects would be expected to be more important for the short than for the long tubes. For most of the cases that follow, the inlet reservoir temperature was set equal to the inlet gas temperature ($t_{r,\,i} = t_{g,\,i}$), and the exit reservoir temperature was set equal to the exit gas temperature ($t_{r,\,e} = t_{g,\,e}$). For cases where this was not done it will be specifically noted.

Effect of emissivity

The effect of wall emissivity on the wall temperature distribution is shown in Fig. 2(a) for typical fixed values of the other parameters. When the wall emissivity is decreased, the radiant-heat transfer becomes less efficient and hence the wall temperatures for pure radiation increase. For pure convection, however, the wall temperatures remain the same, since this solution is independent of emissivity. Then, as

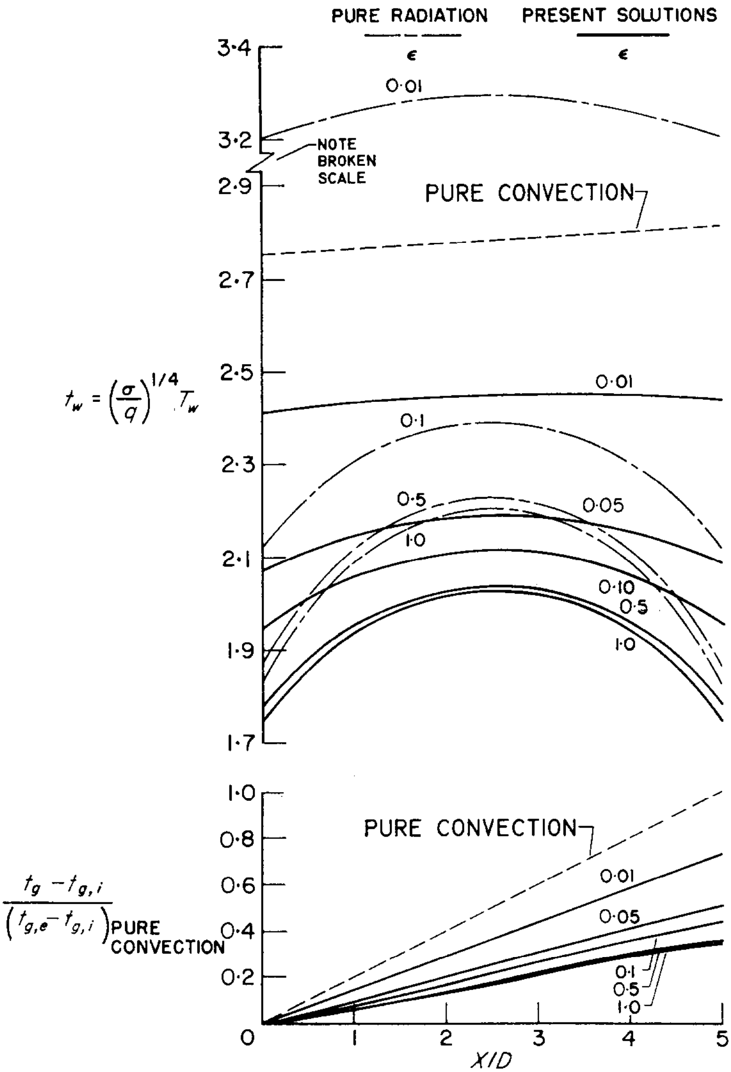

FIG. 2(a). Effect of wall emissivity on temperature distributions in a short tube. $l = 5$, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

discussed in the section on limiting cases, the computed wall temperatures should fall closer to the pure convection curve as the emissivity is decreased. The effect of emissivity on the pure-radiation curves was found to be small for values of ϵ between 1·0 and 0·5, and consequently in this range ϵ has little influence on the solution. Table 1 gives the amounts of heat being transferred by convection to the gas and by radiation to the reservoirs. The radiation loss to the reservoirs causes the wall temperatures to drop off near the ends of the tube. As expected, when ϵ decreases a greater portion of the heat is transferred to the gas. This is also shown in the lower part of Fig. 2(a) where the gas temperature variation approaches the pure convection line as ϵ becomes small.

In Fig. 2(b), the solutions in Fig. 2(a) are

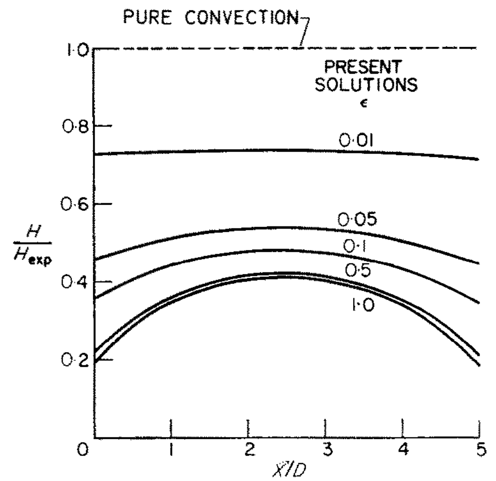

FIG. 2(b). Effect of wall emissivity on ratio of heat-transfer coefficients for pure convection and combined convection and radiation for a short tube. $l = 5$, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

plotted in terms of the radiation correction factor $H/H_{\exp}$, which was discussed previously. Even for an emissivity as low as 0·01 the correction factor is quite large and the effect of radiation cannot be ignored when compared to the convection effects.

Effect of Stanton number

In Fig. 3, the results for different values of Stanton number S are plotted for $\epsilon = 1$ and 0·01. For the same convected heat transfer, an increase in the Stanton number tends to increase the axial temperature gradient along the tube. The present results show that this parameter does not have a significant effect over a physically realistic range of values. Since the changes with S are not large, the influence of ϵ can be shown for this entire range of S by choice of a typical S-value and giving results for several ϵ. This is given in Fig. 2(a) where $S = 0{\cdot}01$ is chosen. This is a reasonable value as shown by

Table 1. Fractional heat losses by convection and radiation for a short tube, $l = 5$

		ϵ	$t_{r,e}$	Convection out/ heat in	Radiation to inlet reservoir/ heat in	Radiation to outlet reservoir/ heat in
Effect of ϵ		0·01	$1{\cdot}546 = t_{g,e}$	0·734	0·136	0·130
($H = 0{\cdot}8$, $S = 0{\cdot}01$,		0·1	$1{\cdot}528 = t_{g,e}$	0·440	0·282	0·278
$t_{r,i} = t_{g,i} = 1{\cdot}5$)		1	$1{\cdot}522 = t_{g,e}$	0·345	0·329	0·326
Effect of S	$S = 0{\cdot}005$	0·01	$1{\cdot}523 = t_{g,e}$	0·736	0·133	0·131
($H = 0{\cdot}8$,	0·005	1	$1{\cdot}511 = t_{g,e}$	0·346	0·328	0·326
$t_{r,i} = t_{g,i} = 1{\cdot}5$)	0·02	0·01	$1{\cdot}591 = t_{g,e}$	0·728	0·144	0·128
	0·02	1	$1{\cdot}543 = t_{g,e}$	0·341	0·333	0·326
Effect of H	$H = 0{\cdot}2$	0·01	$1{\cdot}575 = t_{g,e}$	0·300	0·357	0·343
($S = 0{\cdot}01$,	0·2	1	$1{\cdot}527 = t_{g,e}$	0·108	0·449	0·443
$t_{r,i} = t_{g,i} = 1{\cdot}5$)	1·5	0·01	$1{\cdot}529 = t_{g,e}$	0·873	0·065	0·062
	1·5	1	$1{\cdot}517 = t_{g,e}$	0·513	0·245	0·242
Effect of $t_{g,i}$	$t_{g,i} = 0{\cdot}5$	0·01	$0{\cdot}558 = t_{g,e}$	0·928	0·035	0·037
($H = 0{\cdot}8$, $S = 0{\cdot}01$,	0·5	1	$0{\cdot}544 = t_{g,e}$	0·708	0·144	0·148
$t_{r,i} = t_{g,i}$)	3·0	0·01	$3{\cdot}025 = t_{g,e}$	0·394	0·323	0·283
	3·0	1	$3{\cdot}006 = t_{g,e}$	0·090	0·459	0·451
Effect of $t_{r,e}$		0·01	0	0·719	0·095	0·186
($H = 0{\cdot}8$, $S = 0{\cdot}01$,		1	0	0·287	0·306	0·407
$t_{r,i} = t_{g,i} = 1{\cdot}5$)		0·01	5	1·782	4·861	− 5·643
		1	5	1·993	4·767	− 5·760

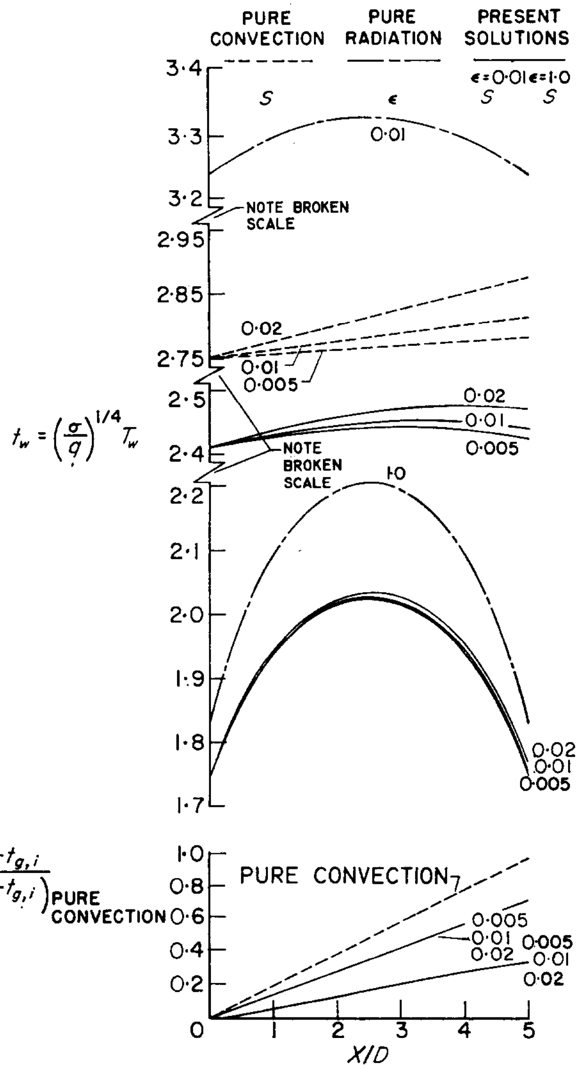

FIG. 3. Effect of Stanton number S on temperature distributions in a short tube. $l = 5$, $H = 0{\cdot}8$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

use of the formula for fully developed turbulent pipe flow, $Nu = 0{\cdot}023\ Re^{0{\cdot}8}\ Pr^{0{\cdot}4}$. The value of S for $Re = 100\,000$ and $Pr = 0{\cdot}7$ is about 0·011.

Effect of heat-transfer coefficient

The results for different values of H are shown in Fig. 4 for emissivities of 1 and 0·01. If q is constant, a variation of H corresponds to a proportional variation in the heat-transfer coefficient h. Since the parameter S, which also contains h, is being kept constant in Fig. 4, this implies a similar proportional variation in $\rho u_m c_p$ to keep S constant while H is varied. It was shown by Fig. 3 that a variation of $\rho u_m c_p$ as contained in S does not have a large effect on the results for h held constant, so that Fig. 4 primarily indicates the effect of different h-values.

For a small H, the convection is poor and the

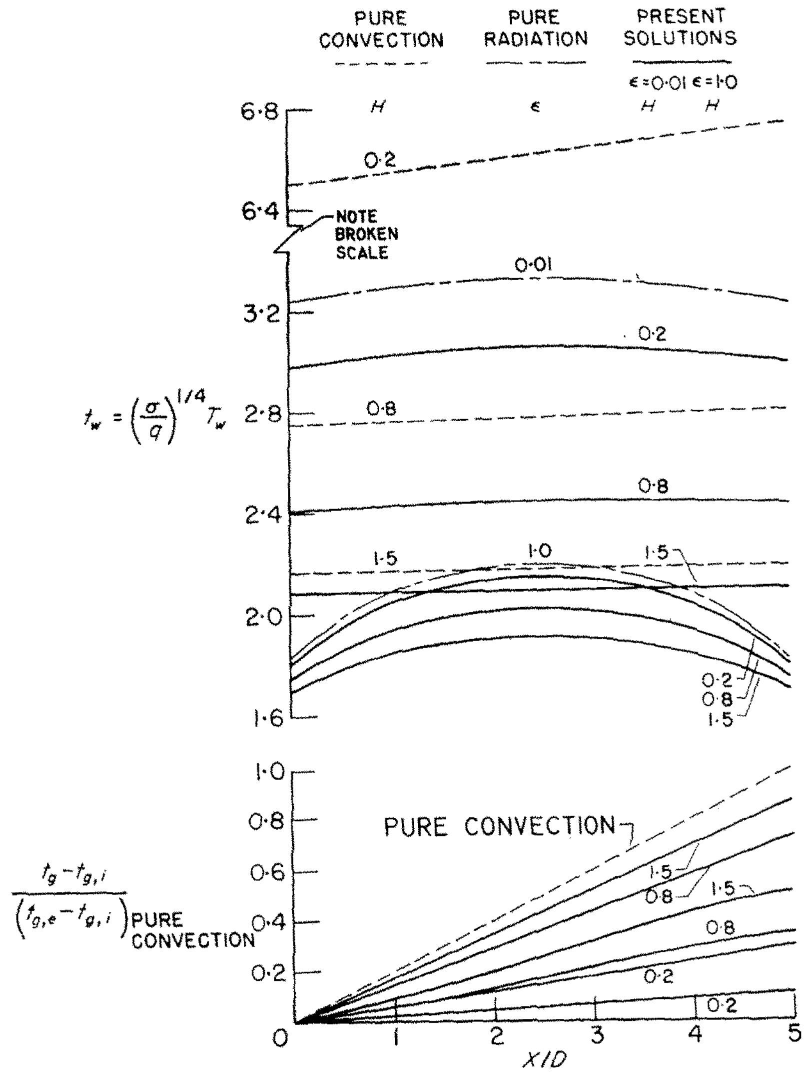

FIG. 4. Effect of dimensionless convective heat-transfer coefficient H on temperature distributions in a short tube. $l = 5$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

pure-convection solution gives high wall temperatures. For $H = 0{\cdot}2$ the pure convection result is higher than the pure radiation curve for $\epsilon = 0{\cdot}01$, so even for this low emissivity the heat can leave the tube more efficiently by radiation. Hence the solution lies below the pure radiation curve. For $H = 1{\cdot}5$, however, the pure-convection curve gives much lower temperatures than the pure radiation solution for $\epsilon = 0{\cdot}01$, and the solution lies slightly below the convection results. For $H = 0{\cdot}8$, the details on the effect of emissivity are given in Fig. 2.

Effect of inlet gas temperature

Figure 5 shows the effect of varying the inlet gas temperature in a duct of length 5. As the temperature is raised, the radiation exchanges become more important and the solutions move toward the pure-radiation curves. The effect of

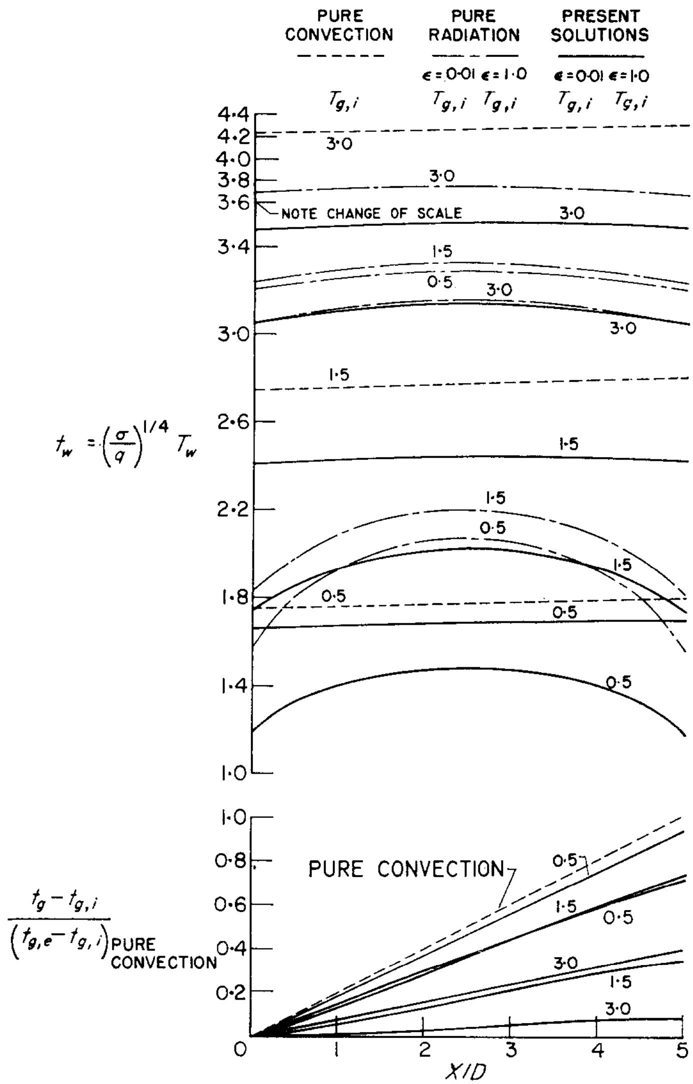

FIG. 5. Effect of inlet-gas temperature $t_{g,i}$ on temperature distributions in a short tube. $l = 5$, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i}$, $t_{r,e} = t_{g,e}$.

wall emissivity is shown in detail in Fig. 2 for $t_{g,i}$ equal to 1·5.

Effect of exit reservoir temperature

For the preceding cases the exit reservoir temperature was set equal to the exit gas temperature. In Fig. 6 the exit reservoir temperature was set at different values independent of the exit gas temperature. The results demonstrate the large influence that the exit reservoir has on the tube-wall temperature distribution. Table 1 shows that in some cases the heat radiated into the tube is as large as the heat flux imposed at the wall. If the inlet reservoir were heated it would be expected to produce the same type of effect near the tube entrance. Hence in this instance, for a rocket engine where the nozzle walls can see the combustion chamber,

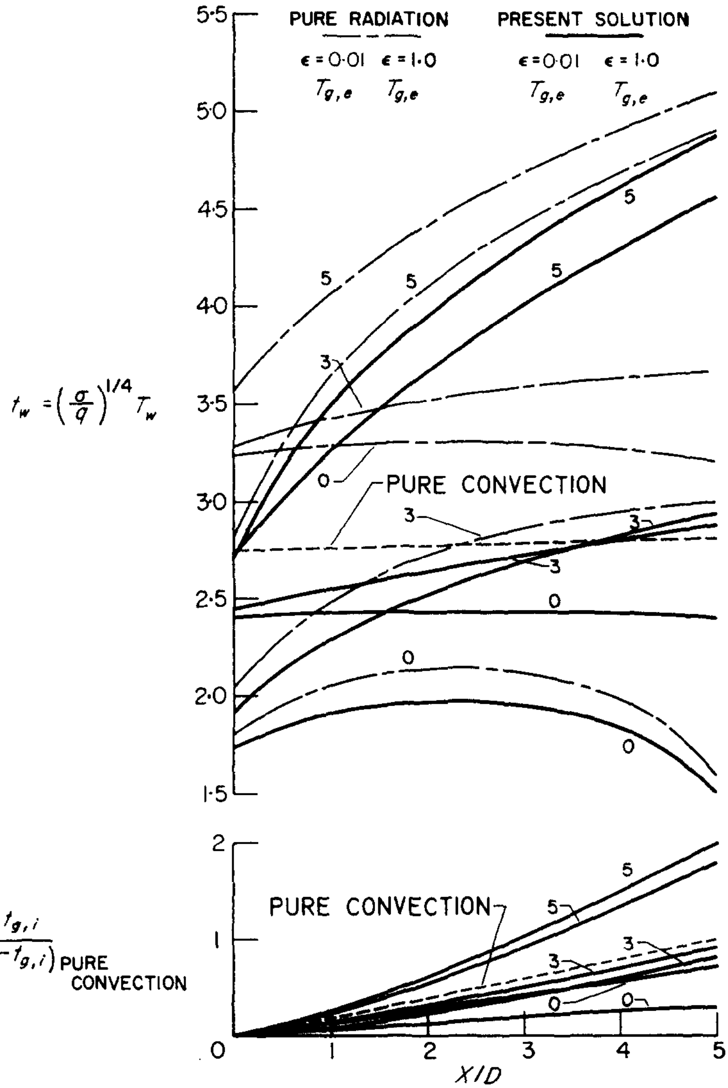

FIG. 6. Effect of exit-reservoir temperature $t_{r,e}$ on temperature distributions in a short tube. $l = 5$, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$.

this indicates that the heat load on the wall may be quite large because of the radiant interchange.

RESULTS FOR A LONG TUBE ($L/D = 50$)

Results will now be given for a long tube with a length–diameter ratio of 50. In the central portion of the tube the wall temperatures fall close to the pure-convection solution. This shows that the net radiation exchange is very small even for $\epsilon = 1$, and hence in this region heat is transferred mainly by convection. Near the ends of the tube the wall can readily exchange heat with the reservoirs, and hence in these regions the net radiation exchange can be quite large. This causes the wall temperature near the ends to be close to the pure-radiation solution.

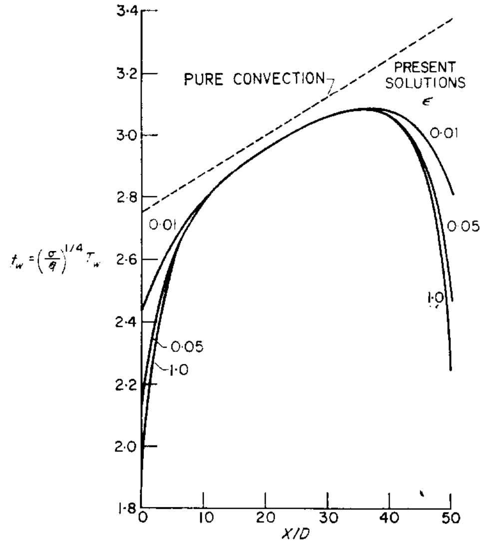

FIG. 7(a). Effect of wall emissivity ϵ on the temperature distributions in a long tube. $l = 50$, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

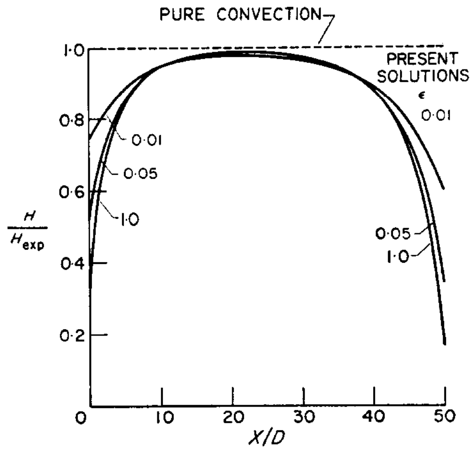

FIG. 7(b). Effect of wall emissivity on ratio of heat-transfer coefficients for pure convection and combined convection and radiation for a long tube. $l = 50$, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

Table 2. Fractional heat losses by convection and radiation for a long tube, $l = 50$

		ϵ	$t_{r,e}$	Convection out/ heat in	Radiation to inlet reservoir/ heat in	Radiation to outlet reservoir/ heat in
Effect of ϵ		0·01	$2{\cdot}066 = t_{g,e}$	0·906	0·039	0·055
($H = 0{\cdot}8$, $S = 0{\cdot}01$,		0·1	$2{\cdot}047 = t_{g,e}$	0·875	0·053	0·072
$t_{r,i} = t_{g,i} = 1{\cdot}5$)		1	$2{\cdot}043 = t_{g,e}$	0·869	0·056	0·075
Effect of S	$S = 0{\cdot}01$	0·01	$2{\cdot}066 = t_{g,e}$	0·906	0·039	0·055
($H = 0{\cdot}8$,	0·01	1	$2{\cdot}043 = t_{g,e}$	0·869	0·056	0·075
$t_{r,i} = t_{g,i} = 1{\cdot}5$)	0·02	0·01	$2{\cdot}061 = t_{g,e}$	0·881	0·044	0·075
	0·02	1	$2{\cdot}556 = t_{g,e}$	0·845	0·059	0·096
Effect of H	$H = 0{\cdot}2$	0·01	$2{\cdot}743 = t_{g,e}$	0·497	0·240	0·263
($S = 0{\cdot}01$,	0·2	1	$2{\cdot}691 = t_{g,e}$	0·476	0·251	0·273
$t_{r,i} = t_{g,i} = 1{\cdot}5$)	0·8	0·01	$2{\cdot}066 = t_{g,e}$	0·906	0·039	0·055
	0·8	1	$2{\cdot}043 = t_{g,e}$	0·869	0·056	0·075
Effect of $t_{g,i}$	$t_{g,i} = 1{\cdot}5$	0·01	$2{\cdot}066 = t_{g,e}$	0·906	0·039	0·055
($H = 0{\cdot}8$, $S = 0{\cdot}01$,	1·5	1	$2{\cdot}043 = t_{g,e}$	0·869	0·056	0·075
$t_{r,i} = t_{g,i}$)	3	0·01	$3{\cdot}467 = t_{g,e}$	0·748	0·122	0·130
	3	1	$3{\cdot}444 = t_{g,e}$	0·705	0·141	0·154
Effect of $t_{r,e}$		0·01	0	0·895	0·039	0·066
($H = 0{\cdot}8$, $S = 0{\cdot}01$,		1	0	0·851	0·056	0·093
$t_{r,i} = t_{g,i} = 1{\cdot}5$)		0·01	4·5	1·108	0·040	− 0·148
		1	4·5	1·116	0·056	− 0·172

Effect of emissivity

The effect of wall emissivity is shown in Fig. 7(a). The emissivity has an effect only near the inlet and outlet reservoirs. As emissivity is decreased, heat cannot be radiated as efficiently and the wall temperatures tend toward the pure-convection solution. In the central part of the tube, heat is transferred mostly by convection. The small amount which is transferred by radiation is independent of the emissivity. This is because the radiation for a small emissivity is reflected and re-reflected from the walls many times before it can escape to the ends of the tube. The total effect of all the reflections causes the radiation exchange to behave as if the walls were black. Table 2 shows the amounts of heat being radiated or convected from the tube. For long tubes most of the heat is transferred by convection.

In Fig. 7(b) the radiation correction factor is plotted for the same cases given in Fig. 7(a). In the central region the correction factor is close

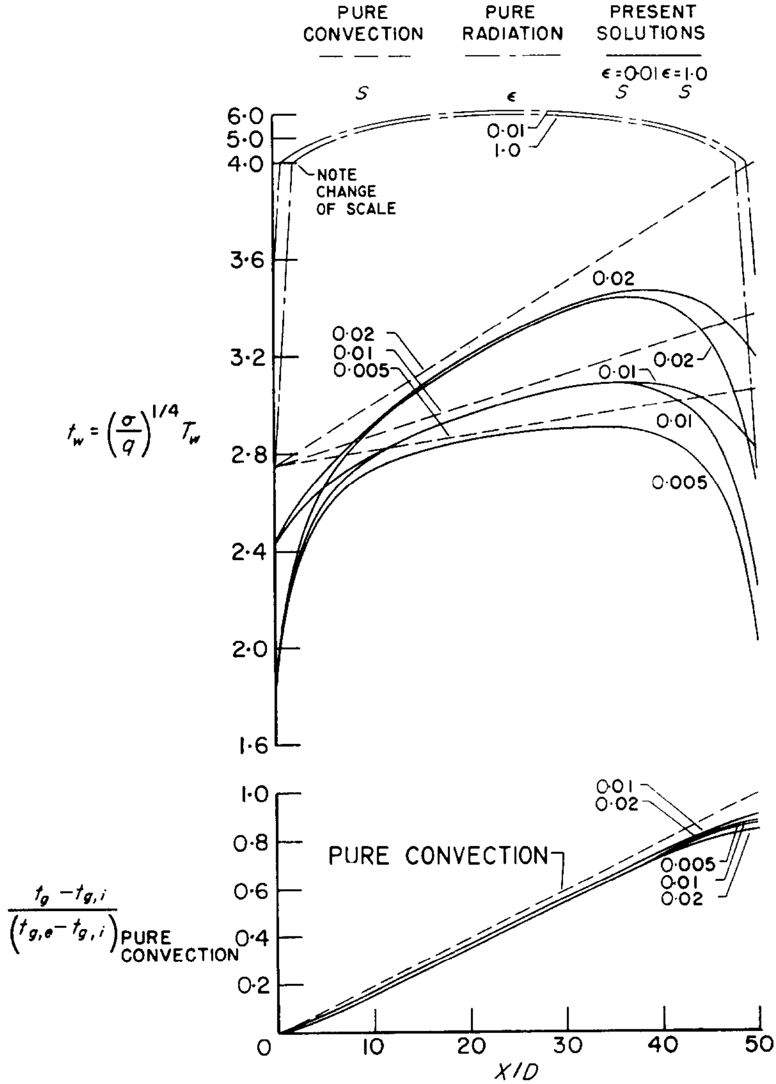

FIG. 8. Effect of Stanton number S on temperature distributions in a long tube. $l = 50$, $H = 0{\cdot}8$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

to unity so that, in an experiment, the convective heat-transfer coefficient could be determined directly without a significant radiation error.

Effect of Stanton number

The effect of Stanton number S is shown in Fig. 8 for emissivities of 0·01 and 1. The temperatures obtained for pure radiation are quite high since it is difficult for the heat to dissipate from the central portion of the tube to the end reservoirs. As a result, most of the heat leaves by convection which is the more efficient heat-transfer mechanism and the solutions are convection dominated. Since the pure-convection results are quite dependent on the parameter. S, the same dependence is present in the solutions. The gas temperature curves in the lower part of the figure are close to the pure convection line owing to the small radiation losses.

Effect of convective heat-transfer coefficient

Figure. 9 shows the effect of the dimensionless heat-transfer coefficient H on the temperature distribution. In the central part of the tube, for

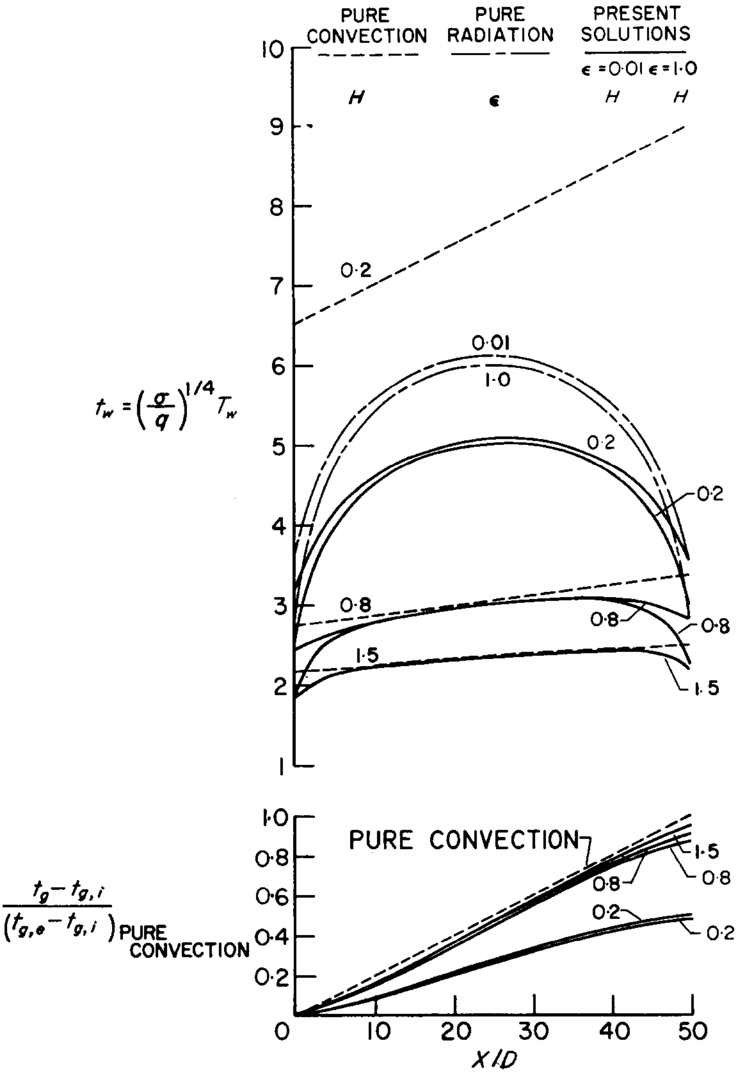

FIG. 9. Effect of dimensionless convective heat-transfer coefficient H, on temperature distributions in a long tube. $l = 50$, $S = 0·01$, $t_{r,i} = t_{g,i} = 1·5$, $t_{r,e} = t_{g,e}$.

large values of H the wall temperatures for pure convection are much lower than for pure radiation. As a result, practically all the heat is transferred by convection in this region, giving wall temperatures only slightly below the pure-convection curve. The ends of the tube, however, can see the colder environment quite well, and in this region heat is transferred by the combined process of convection and radiation, giving temperatures below the results for either process alone. For a small H of 0·2 the wall temperature distribution for pure convection is higher than the pure-radiation curve; hence the heat can leave more efficiently by radiation which gives a solution that is strongly radiation dominated.

Effect of inlet gas temperature

Temperature distributions for a tube of length $L/D = 50$ and for various inlet gas temperatures are given in Fig. 10. As the temperature level is raised the radiation effects become more

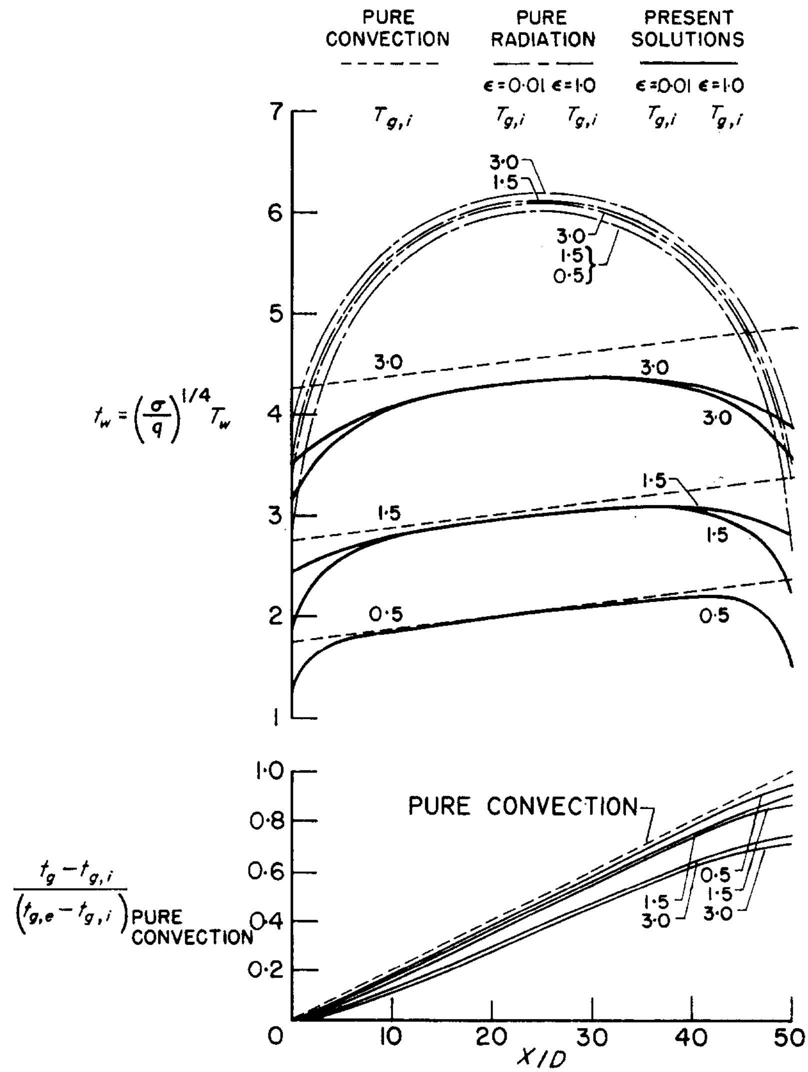

FIG. 10. Effect of inlet gas temperature $t_{g,i}$ on temperature distributions in a long tube. $l = 50$, $H = 0·8$, $S = 0·01$, $t_{r,i} = t_{g,i}$, $t_{r,e} = t_{g,e}$.

important and the wall temperature distribution has a slightly greater deviation from the results for pure convection. When the emissivity is decreased, the direct radiation losses to the reservoirs are smalller and the wall temperatures near the ends of the tube are not as low as they were for $\epsilon = 1$.

Effect of exit reservoir temperature

For each curve in Fig. 11, the exit reservoir has been maintained at a fixed temperature rather than being set equal to the exit gas temperature. When the exit reservoir is at a high temperature, heat is radiated into the tube from the reservoir which elevates the tube wall temperatures near the exit end. When the emissivity is decreased, the radiation exchanges are reduced and the values move toward the pure convection curve. Since, for $L/D = 50$, the exit of the tube is far from the inlet, the temperature

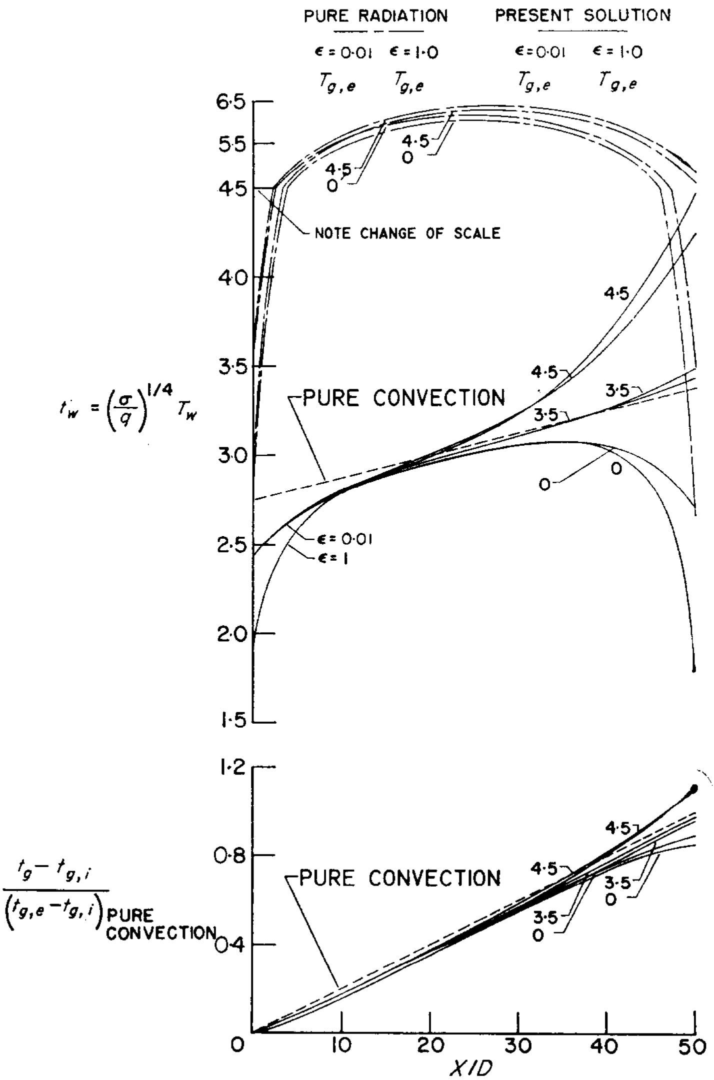

FIG. 11. Effect of exit reservoir temperature $t_{r,e}$ on temperature distributions in a long tube. $l = 50$, $H = 0·8$, $S = 0·01$, $t_{r,i} = t_{g,i} = 1·5$.

of the exit reservoir has no influence on the solution near the tube entrance.

EFFECT OF TUBE LENGTH–DIAMETER RATIO

In the previous sections, temperature distributions have been given for short and long tubes with length–diameter ratios of 5 and 50. Now, as shown in Fig. 12, a few solutions are given for tubes with L/D equal to 10, 20 and 30, so that results can be interpolated for tubes with other L/D. As discussed previously, convection becomes more important as the tube length is increased, and hence the curves move toward the pure-convection results as L/D becomes larger. For a low emissivity the curves are even closer to the pure-convection results. This is also demonstrated in Table 3 where numerical results for several emissivities are given.

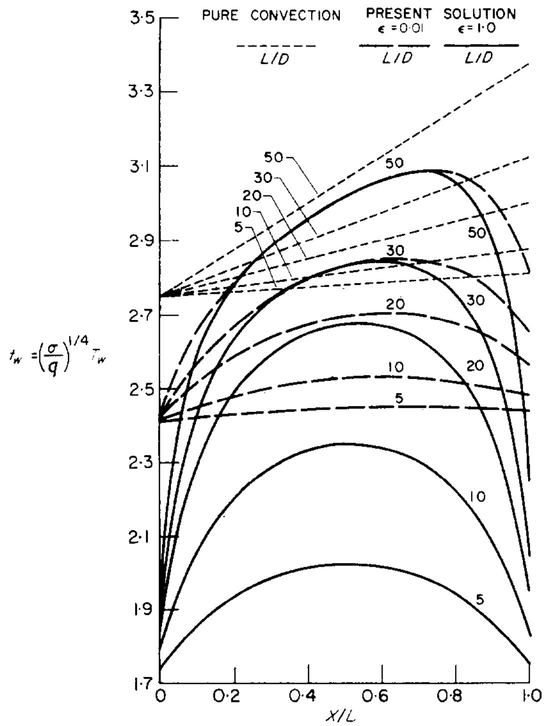

FIG. 12. Effect of tube length on wall temperature distributions. $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{r,i} = t_{g,i} = 1{\cdot}5$, $t_{r,e} = t_{g,e}$.

CONCLUDING REMARKS

The purpose of this report has been principally to examine the effect of wall emissivity on combined radiation and forced convection in a tube. The inside of the tube wall was assumed to be a diffuse gray surface and the gas flowing through the tube is transparent to radiation. The convective heat-transfer coefficient between the tube wall and the gas was assumed constant. In the thermal-entrance region this would not be true, since the heat-transfer coefficient would decrease with X, from a high value close to the tube entrance to the fully developed value which, for turbulent gas flow, is reached about 12 diameters down the tube. The high convection coefficients at small X would decrease the wall temperatures and hence lessen the radiation effects. An indication of the magnitude of this effect can be obtained by following the reasoning given at the end of the section on limiting cases, although a detailed study would be a subject for future analysis.

For long tubes with an L/D of about 50, the emissivity had very little effect in the central region of the tube. In this region, convection accounts for almost all of the heat transfer, and radiation is relatively unimportant. The heat radiated in the central part of the tube undergoes multiple reflections between surface elements before escaping to the ends of the tube. This tends to produce the same net exchange as black radiation from the wall, which further reduces the influence of emissivity. Near the inlet and exit of a long tube the radiation exchanges are reduced when the emissivity is decreased, since the direct radiation to the reservoirs at the ends of the tube is smaller. For very short tubes this direct radiation is important throughout the entire tube length, and hence the wall emissivity has a substantial influence on the entire temperature distribution.

Some consideration was given to the interpretation of forced-convection experiments with electrically heated tubes for measurement of convective heat-transfer coefficients at high temperatures. When there is a large radiation exchange in the tube, the measured heat-transfer coefficients must be corrected to yield results for convection alone. The magnitude of the radiation effect was demonstrated by presentation of some

Table 3. Effect of tube length on fractional heat losses by convection and radiation, $H = 0{\cdot}8$, $S = 0{\cdot}01$, $t_{g,i} = t_{r,i} = 1{\cdot}5$

	L	ϵ			
		0·01	0·05	0·1	1
Convection out/heat in	10	0·764	0·619	0·582	0·538
	20	0·824	0·755	0·740	0·722
	50	0·906	0·881	0·875	0·869
Radiation to inlet reservoir/heat in	10	0·119	0·191	0·209	0·231
	20	0·086	0·119	0·126	0·135
	50	0·039	0·050	0·053	0·056
Radiation to outlet reservoir/heat in	10	0·117	0·190	0·209	0·231
	20	0·090	0·126	0·134	0·143
	50	0·055	0·069	0·072	0·075
Exit-reservoir temperature ($t_{r,e} = t_{g,e}$)	10	1·595	1·577	1·573	1·567
	20	1·706	1·689	1·685	1·681
	50	2·066	2·051	2·047	2·043

of the solutions in terms of the ratio H/H_{exp}, which is the heat-transfer coefficient for convection alone divided by the coefficient for combined radiation and convection. This correction factor is reduced when the emissivity of the surface is decreased. However, the results in, for example, Fig. 2(b) show that the effect of emissivity is very slight until ϵ becomes smaller than about 0·5, so in some instances small emissivities are required to reduce the radiation exchange substantially.

Another way to reduce the radiation exchange is to control the temperatures in the reservoirs at the ends of the tube. If the reservoir temperatures are raised to the proper values, very little heat will be exchanged with them and practically all of the heat supplied at the tube wall will leave by convection. A few solutions were obtained where the inlet and exit reservoirs were maintained respectively at the wall temperatures computed from equation (23) for convection alone at $X = 0$ and $X = L$. The results gave wall and gas temperatures that were within a few tenths of a per cent agreement with the pure-convection solution, so that the net radiation effect was eliminated. Under these conditions the convection coefficient could be measured directly in a tube without the need for applying a radiation correction.

ACKNOWLEDGEMENT

The authors would like to thank Miss Eileen M. Norris who wrote the programs for the digital computer and plotted the results for the solutions given here.

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Résumé—Cet article présente une étude analytique de l'influence du rayonnement sur les échanges par convection dans une conduite circulaire. On impose à la paroi du tube un flux de chaleur constant à l'aide d'un dipositif extérieur (chauffage électrique constant par exemple), la distribution de température à la paroi est donnée par le calcul. Le gaz qui s'écoule dans le tube laisse passer le rayonnement et ne modifie pas, par suite, les échanges par rayonnement entre les éléments de le paroi interne de la conduite. On suppose que l'intérieur de la conduite est une surface grise diffuse et que l'extérieur est parfaitement isolé. Les solutions dépendent de sept paramètres indépendants tels que: émissivité de la paroi, température d'entrée du gaz, rapport longueur/diamètre du tube. Des exemples numériques montrent l'influence de ces paramètres et la façon dont le rayonnement modifie la distribution de température à la paroi qui existe lorsqu'il n'y a que de la convection.

Zusammenfassung—Der Einfluss der Wärmestrahlung bei gleichzeitigem konvektivem Wärmeübergang wird für ein zylindrisches Rohr analytisch untersucht. Mit einer gleichmässig gewickelten elektrischen Heizung liess sich eine konstante Wärmestromdichte durch die Wand erreichen; die Temperaturverteilung an der Wand lieferte die Analyse. Das im Rohr strömende Gas ist strahlungsdurchlässig und stört somit den Strahlungsaustausch zwischen den Flächenelementen der Rohrinnenseite nicht. Diese Rohrinnenseite wird als diffus strahlende graue Fläche angenommen; die Rohraussenseite sei adiabat isoliert. Sieben voneinander unabhängige Parameter wie z.B. das Emissionsverhältnis der Wand, die Gaseintrittstemperatur, das Längendurchmesserverhältnis des Rohres kennzeichnen die Lösung. Zahlenbeispiele zeigen sowohl den Einfluss dieser Parameter wie auch die Temperaturverteilung an der Wand bei Wärmestrahlung gegenüber jener bei Konvektion allein.

Аннотация—Приводится аналитическое рассмотрение влияния лучистого теплообмена на одновременно происходящий внутри круглой трубы конвективный перенос тепла. Принимается, что через стенку турбы проходит постоянный тепловой поток, создаваемый с помощью какого-либо внешнего средства, например, постоянным нагревом электрическим током. Аналитически находится распределение температуры на стенке. Газ, протекающий в трурбе, прозрачен и поэтому не оказывает влияния на лучистый теплообмен между элементами внутренней поверхности трубы. Принято, что внутренняя сторона поверхности трубы является рассеивающей серой поверхностью, а наружная сторона хорошо изолирована. Решение даётся как функция следующих независимых параметров: коэффициента лучеиспускания стенки, температуры газа на входе и отношения длины трубы к её диаметру. Приводятся численные решения, из которых видно влияние этих параметров на процесс теплообмена. Решения показывают, как излучение изменяет распределение температур на стенке, сложившееся под воздействием одной конвекции.